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SEISMIC RELIABILITY ANALYSIS OF BASE-ISOLATED BUILDINGS

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ABSTRACT

Seismic base-isolation is achieved via inserting flexible isolator element between the foundation and superstructure which lengthens the vibration period and increases the energy dissipation. Values adopted for uncertain parameters in earthquake engineering deviate from their nominal values. The stochastic response analysis of the base-isolated building is studied considering uncertainties in the characteristics of the earthquakes. Artificial acceleration time histories developed using probabilistic ground motion model are used in the study. The uncertainties in peak ground acceleration, frequency content and time duration are considered to develop 5000 artificial acceleration time-histories. A bilinear model of the isolator described by its characteristic strength, post-yield stiffness, and yield displacement is used and the stochastic response is calculated by using an ensemble of generated earthquakes.The study also presents the results of seismic reliability analysis of the five-storey base-isolated building. The first order reliability method (FORM) and Monte Carlo simulation (MCS) method are used to evaluate the probability of failure associated with the top floor acceleration response of the building. The top floor acceleration response statistics are used in the performance function considering 0.3g as a maximum allowable acceleration and the probability of failure is evaluated. The probability of failure evaluated using the FORM is fairly in good agreement with the value obtained by the MCS.

Keywords: Base isolation, Artificialearthquakes, Stochastic response, Reliability, FORM, MCS.

INTRODUCTION

Ground motion at a particular site due to earthquakes is influenced by source, travel path and local site conditions. The first relates to the size and source of the source mechanism of the earthquake. The second describes the path effects of the earth as the wave travels from the source to the site. The third describes the effects of the upper hundreds of meters of rocks and soil and the surface topography at the site. It is well known that earthquake ground motions are nonstationary in time and frequency domains. Temporal nonstationarity refers to the variation in the intensity of the ground motion in time. Spectral nonstationarity refers to the variation in the frequency content of the motion in time.

One of the emerging tools for protecting structures from the damaging effects of earthquake ground motion is the use of isolation systems.Base isolation or base isolation system, is a collection of structural elements which substantially decouple a superstructure from its substructure which is resting on a shaking ground, thus protecting a building or non-building structure's integrity. The base isolation minimizes the inter-storey deformations and the floor accelerations by interposing elements of high axial and low horizontal stiffness between the structure and the foundation. It is meant to enable a building or non-building structure to survive a potentially devastating seismic impact through a proper initial design or subsequent modifications. In some cases, application of base isolation can raise both a structure's seismic performance and its seismic sustainability considerably. Various theories of seismic isolation, testing programmes and isolation systems have been used in structures (Kelly, 1986; Jangid and Datta, 1995). Previous studies have demonstrated that the Monte Carlo simulation is an effective method in obtaining stochastic response statistics.The objective of this study is to carry out the stochastic response evaluation and reliability analysis of the isolated building structure under earthquake excitations with emphasis on the uncertainty in the earthquake loading.The seismic reliability analysis of the five-storey base-isolated building is analysed using the first order reliability method (FORM) and Monte Carlo simulation (MCS) method. The structural characteristics are considered as deterministic and the top floor acceleration response is evaluated using 5000 artificial acceleration time histories. Using these response values, probabilistic characteristics of the top floor acceleration response are obtained. These characteristics are used in the performance function considering 0.3*g* as a maximum allowable acceleration and the probability of failure is evaluated. All the 5000 acceleration response values of the top floor are used in the MCS to obtain the probability of failure associated with the five-storey base-isolated building. The probability of failure evaluated using the FORM is fairly in good agreement with the value obtained by the MCS.

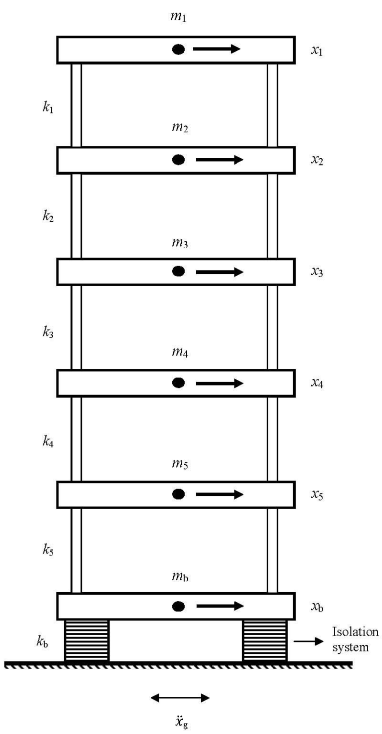

Fig. 1 Mathematical model of a five storey base-isolated building

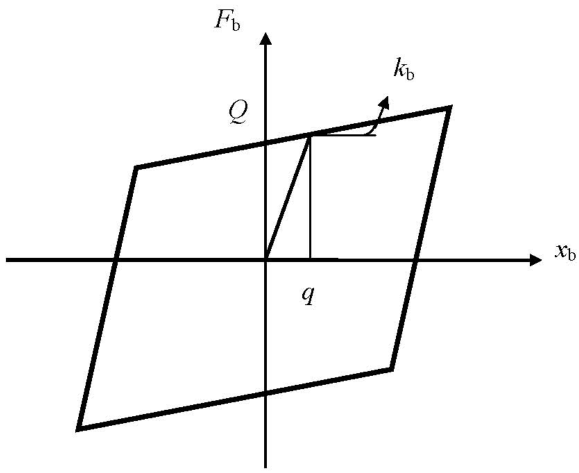

Fig. 2 Mathematical model of the isolator

DETERMINISTIC RESPONSE OF AN ISOLATED BUILDING

In this study, the lumped mass modelling is adopted for the superstructure and the isolator. The effect of rotation in the structure and isolator is not taken into consideration. The base-isolated building is modelled as a shear type structure mounted on isolation system with one lateral degree-of-freedom at each floor. Figure1 shows the idealized mathematical model of the five-storey base-isolated building considered in the study. The following assumptions are made: (i) The superstructure is considered to remain within the elastic limit during the earthquake excitation; (ii) The floors are assumed as rigid in its own plane and the mass is supposed to be lumped at each floor level; (iii) The columns are inextensible and weightless providing the lateral stiffness; (iv) The system is subjected to single horizontal component of the earthquake ground motion; (v) The effect of soil-structure interaction is neglected. The $x_j$is the relative floor displacement at the j^{th} floor, m_j is the floor mass at the j^{th} floor, k_j is the stiffness of the j^{th} floor, x_b is the displacement of the isolator and m_b is the mass of the isolator.

The force-deformation behaviour of the isolator is modelled as nonlinear hysteretic represented by the bilinear model. This model is depicted in Fig. 2. The nonlinear force-deformation behaviour of the isolation system is modelled through the bilinear hysteresis loop characterized by three parameters: (i) characteristic strength, Q; (ii) post-yield stiffness, k_b; and (iii) yield displacement, q. The characteristic strength, Q is related to the yield strength of the lead core in the elastomeric bearings and friction coefficient of the sliding type isolation systems. The post yield stiffness of the isolation system, k_b is generally designed in such a way to provide the specific value of the isolation period, T_b expressed as

(1)

where is the total mass of the base-isolated structure. The characteristic strength, Q is mathematically related to the damping ratio, ζ_b by the following equation (Naeim and Kelly, 1999).

(2)

where D is the design displacement. Thus, the bilinear hysteretic model of the base isolation system can be characterized by specifying the three parameters namely T_b, Q and q.

Equations of Motion and Solution Procedure

The general equations of motion for the superstructure-isolator model illustrated in Fig. 1 can be expressed as

(3)

where is the column vector of relative structural displacements with respect to the isolator, is the column vector of total structural displacements, M is the mass matrix of the structure, C is the viscous damping matrix of the structure, and K is thestiffness matrix of the structure. The is expressed as

(4)

where x_g is the displacement of the ground due to the earthquake, x_b is the displacement of the isolator.The equations of motion for a five-storey base-isolated building is written as

(5)

where

(6)

(7)

(8)

(9)

The damping matrix of the superstructure, C is not known explicitly. It is constructed by assuming the modal damping ratio in each mode of vibration of the superstructure.

Classical modal superposition technique cannot be employed in the solution of equations of motion here, because: (i) the system is non-classically damped because of the difference in the damping in isolation system compared to the damping in the superstructure and (ii) the force-deformation behaviour of the isolation systems considered is nonlinear. Therefore, the equations of motion are solved numerically using Newmark's method of step-by-step integration, adopting linear variation of acceleration over a small time interval of Δt. The response quantities of interest such as acceleration, velocity and displacement at any degree of freedom and force in the isolator are calculated at each time interval (Wen, 1976).

Numerical Example

In order to find the deterministic response of the isolated structure, a recorded earthquake accelerogram is considered. The earthquake motion selected for the study is N00E component of 1989 Loma Prieta earthquake recorded at Los Gatos Presentation Center.The peak ground acceleration (PGA) of the Loma Prieta earthquake is 0.57g. The acceleration response of the structure and the isolator response are calculated. The response quantities of interest are the top floor absolute acceleration and relative isolator displacement.The above response quantities are chosen because the floor accelerations developed in the superstructure are proportional to the forces exerted by the earthquake ground motion. The bearing displacements are crucial in the design of isolation systems. The various parameters of the superstructure and the isolator considered for the example problem aregiven. The ratio of mass of each floor is 1:1:1:1:1 and ratio of stiffness of each floor is 2:3:4:5:6. The floor mass of each floor of the superstructure is considered as equal. The stiffness is considered in such a way that the top floors are less stiff than the bottom floors. The stiffness increases proportionally from top to bottom. The appropriate time period of the five-storey building is taken as 0.5s. The damping ratio of the superstructure is taken as 0.02 and kept constant for all modes of vibration. The inter-storey stiffness of the superstructure is adjusted such that a specified fundamental time period of the superstructure, T_s is achieved. The mass of the isolator is considered to be equal to that of a floor. The damping ratio of the isolator, ζ_b is 0.1. The time period of the isolator, T_b is taken as 2s. The design displacement, D is 53.61 cm and yield displacement, q is taken as 2.5cm. The peak top floor acceleration of the fixed base structure is 2.92g and that of the base isolated structure is 0.66g. The peak isolator displacement is 42.57 cm.

STOCHASTIC SIMULATION OF GROUND MOTIONS

A method to generate an ensemble of artificial earthquake ground motions is described in this section. The method is based on the *probabilistic ground motion model* developed by Jacob et al. (2010). The probabilistic model is based on the fully nonstationary stochastic ground motion model proposed by Rezaeian and Kiureghian (2008) which uses filtering of a discretized white-noise process. The nonstationarity is achieved by modulating the intensity and varying the filter properties in time. The stochastic ground motion model considers both the temporal and spectral nonstationarities. A database to be used in the *probabilistic ground motion model* is created by choosing the recorded earthquake accelerograms. The recorded earthquakes are selected arbitrarily. In the next step, the nine parameters required to depict a particular ground motion are found out for all the 100 ground motions in the database. The probability distributions are created for the parameters of all the earthquake motions. Now the parameters required for the stochastic ground motion model to simulate the artificial ground motions are obtained from these distributions. The Monte Carlo simulationis used to generate an ensemble of ground motions.

The expression to calculate the ground acceleration, at any time t is given as

, (10)

where is the modulating function at time *t* and is the standard random normal variable. A modified version of the Housner and Jennings model (Housner and Jennings 1964) using the modulating function is given as

,
,
,
, (11)

This model has six parameters , , , , and which obey the conditions *, ,* and . The denotes the start time of the process, and denote the start and end times of the strong-motion phase with root mean square (RMS) amplitude,, and and are the parameters that shape the decaying end of the modulating function. The value of is taken as 1and *k*= int where is the time step taken for discretizing the model. The filter function is given as

, (12)

Any damped single- or multi-degree-of-freedom linear system that has differentiable response can be selected as the filter, i.e.,

,
otherwise (13)

which represents the pseudo-acceleration response of a single-degree-of-freedom linear oscillator subjected to a unit impulse, in which denotes the time of the pulse, is the set of parameters of the filter with denoting the natural frequency and denoting the damping ratio, both dependent on the time of application of the pulse. The influences the predominant frequency of the resulting process, whereas influences its bandwidth. The predominant frequency of an earthquake ground motion tends to decay with time. Therefore

(14)

where is the total duration of the ground motion, is the filter frequency at time and is the frequency at time . For a typical ground motion, . Thus, the two parameters and describe the time-varying frequency content of the ground motion.

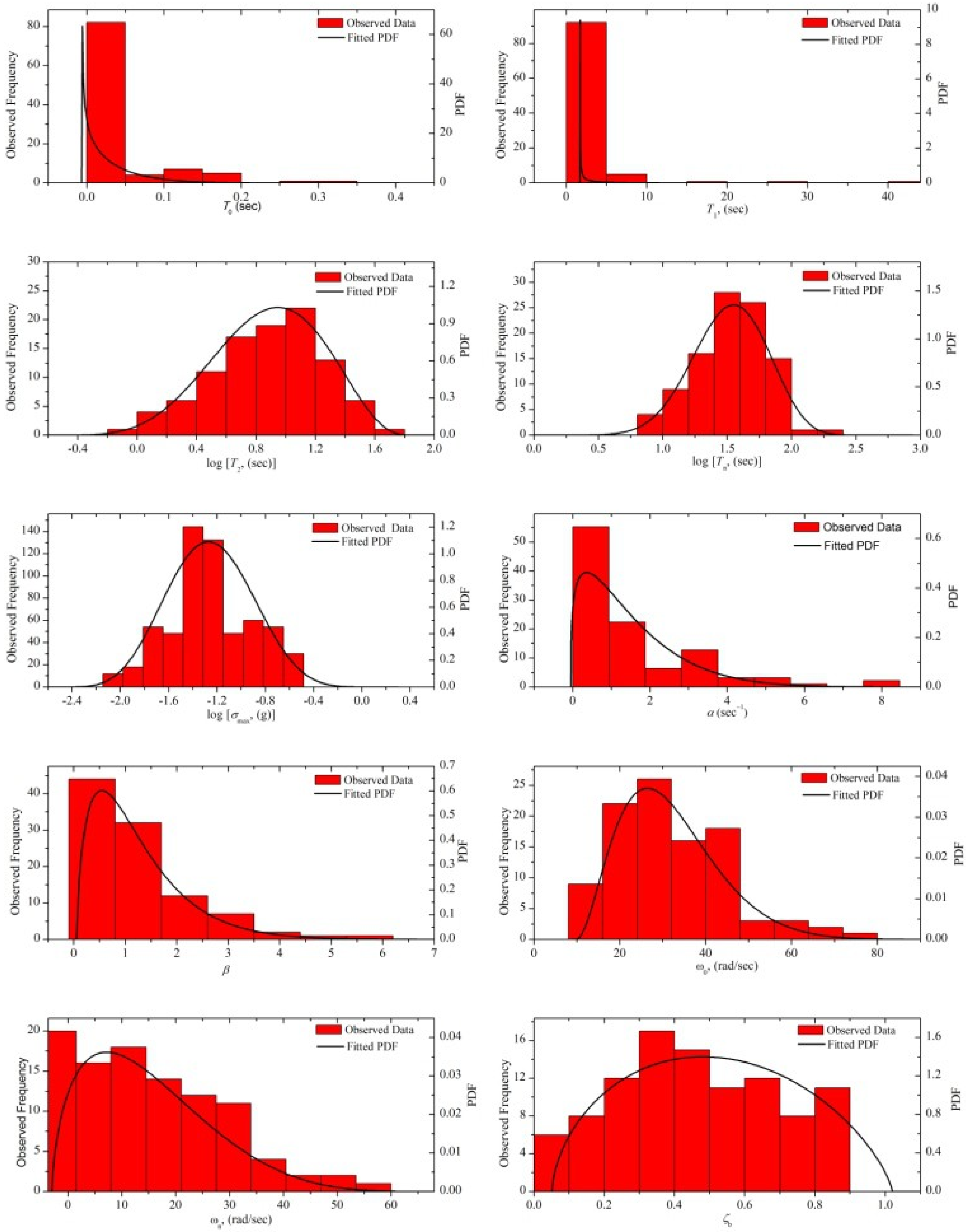

Fig. 3 PDF of parameters superimposed on observed normalized frequencydiagrams

Determination of Model Parameters

The nine parameters of the stochastic ground motion model are found out for each of the earthquakes in the database. The two additional parameters that are required are the time duration T_{n} and the time steps Δt. The T_n is known from the time history and Δt is taken as 0.02 s for all the earthquakes. The six parameters required for the modulating function can be obtained by minimizing the difference between the cumulative energy of the recorded ground motion the cumulative energy of the fitted ground motion. Using the same method of optimization, the parameters in the filter depicting the predominant frequency of the ground motion is obtained by minimizing the difference between the cumulative count of negative maxima and positive minima of the target and the model. The parameters in the filter that depict the bandwidth of the ground motion are obtained by minimizing the difference between the cumulative number of zero-level up-crossings of the target and the model.

Distribution of Model Parameters and Ground Motion Simulation

After identifying the model parameter values by fitting to each of the recorded ground motion in the database, a probability distribution is assigned to the sample of values of each parameter. Now there are 100 sets of parameters representing the 100 earthquakes. The time step Δt is kept constant for all the 100 earthquakes. The distribution models are assigned to each of the 10 parameters. It is found that the data of all the 10 parameters are effectively fitted by β distribution. Figure3shows the normalized frequency diagrams of the fitted model parameters for the entire dataset with the fitted probability density functions (PDFs) superimposed.

A cluster of earthquake ground motions is produced. This is done by randomly selecting the parameters of the stochastic ground motion model. Initially 10000 samples of each parameter are arbitrarily chosen from the distributions of the particular parameter. From this pool of samples, 5000 sets of parameters which satisfy the conditions are chosen. Using these 5000 sets of parameters, 5000 artificial ground motions are generated. This cluster of earthquakes which represents a completely random choice of ground motions are used in the response analysis of the base-isolated building.

STOCHASTIC RESPONSE OF AN ISOLATED BUILDING

The stochastic response of the base-isolated building is evaluated by using direct Monte Carlo simulations. The uncertainty in the characteristics of the ground motion is considered and all the structural input parameters are considered as deterministic. The response quantities of interest are the absolute peak value of the acceleration at the top floor, hereinafter simply referred as top floor acceleration and the peak value of the isolator displacement hereinafter simply referred as isolator displacement. To perform the response analysis, a total of 5000 artificial earthquakes are simulated as described before. Deterministic analysis is performed for each simulation and their corresponding response quantities are evaluated. The deterministic results are then processed to find the peak values, root mean square (RMS) values and distributions of the response quantities. While calculating the responses, the parameters of the structure are kept unchanged. The top floor acceleration is found to be fitted effectively by using β distribution and the isolator displacement is effectively fitted by generalized pareto distribution. The PDFs of the response quantities are plotted and superimposed with their observed frequency diagrams as shown in Fig. 4. The mean and standard deviation of the top floor acceleration are 0.158 and 0.097g respectively. The mean and standard deviation of the isolator displacement are 5.873 and 6.981 cm respectively. For the considered parameters, the extreme top floor acceleration is found to be 0.89g and the extreme isolator displacement is found to be 92cm. Their corresponding RMS values are 0.185g and 9.122 cm.

RELIABILITY ANALYSIS OF THE BASE-ISOLATED BUILDING

The term ‘reliability’ of a structural system may be defined as the probability of satisfactory performance under the given environmental conditions. Past experience shows that apparently conservative designs (based on the traditional ‘factor of safety concept’) are not always safe against failure. Reliability analysis offers a more rational way to evaluate the stability of structural systems by explicitly accounting for uncertainties associated with material properties and loading, environmental conditions and modelling errors.The simplest reliability model involves two resultant variables, called load effect (S) and resistance (R). These resultant variables are usually functions of several random variables $X_1, X_2, \ldots, X_n$. For example, with reference to stability of cantilever retaining wall, S is the net lateral force (causing sliding) and R is the net resisting force. The limit state function associated with the sliding failure can be expressed as

$$M = R - S = g\ (X_1, X_2, \ldots, X_n) \qquad (15)$$

where M is referred to as the safety margin which is a random variable and a function g(X) of the basic variables X_i. The condition g(X) < 0 impliesfailure, while g(X) > 0implies stable behaviour. The boundary, defined by g(X)= 0, separating the stable and unstable states is called the limit state boundary.

Mathematically the probability of failureP_f can be simply defined as follows:

(16)

Where $f_X(x)$ is the joint probability density function of all the basic variables (JPDF). This JPDF may be visualized as a hyper-geometrical space with unit volume, and P_f denotes the function of that volume which lies in the failure domain g(X) < 0.There are several computational difficulties in computing the value of P_f as defined above and hence the problem is numerically addressed by alternative approximate methods, such as Monte Carlo simulation (MCS). Furthermore, the statistical data for the basic variables is generally limited in practice to second order statistics (Mean and standard deviation), and the correlations among the variables are also not well known.In reliability analysis, a popular alternative procedure to address such problems is by the "first order" and "second order" reliability methods (FORM and SORM).

Fig. 4 PDF of response quantities superimposed on observed normalized frequency diagrams

The reliability index can be defined as the distance from the origin (M = 0) to the mean μ_M, measured in standard deviation units. Alternatively, it is the measure of the probability that the safety margin M will be less than zero. Cornell (1969) defined the reliability index as

(17)

If M is a linear function of basic variables that are normally distributed, then M is also normally distributed, whereby the probability of failure is related to reliability index as

$$P_f = \Phi(-\beta) \qquad (18)$$

The mean-value based reliability index has the drawback that its value changes when the limit state function (Eq 15) is expressed as an equivalent, but nonlinear function. For example, $M = R - S$ and $M = (R/S) - 1$ will give different values of β. This problem of 'lack of invariance' was resolved by Hasofer and Lind (1974), by transforming the X variables into an equivalent set of uncorrelated standard unit normal U variables. In this transformed "U – space", the reliability index β is given by the shortest distance from the origin to the surface defining the failure function, g(U). The point of intersection of this line with the failure surface is termed as "design point". The nonlinear failure function can be conveniently approximated by its tangent plane at the "design point", and the corresponding method of finding β is referred to as First Order Reliability Method (FORM).

In the present study, as the available statistics are restricted to second order properties and the basic variables are assumed to be uncorrelated, the following transformation from the X- space to U- space is applicable:

(19)

The problem of computing β by FORM is essentially an optimization problem and many techniques have been developed to achieve this in the literature. The algorithm proposed by Rackwitz and Fiessler (1983) is particularly convenient to apply in FORM with feature of converting non-normal random variables to equivalent normal variables.

Monte Carlo Simulation Method

In the MCS method, each random variable is sampled several times to represent its real distribution according to its probabilistic characteristics. A set of such numbers reflects one possible realization of the problem itself. Solving the problem deterministically for each realization is known as a simulation cycle. Using a large number of simulation cycles enables the generation of the overall probabilistic characteristics of the problem. After repeated simulations, one can assess the sensitivity of the system response to variation in the parameters. Each continuous variable is replaced by a large number of discretevalues generated from the underlying distribution; these valuesare used to compute a large number of values of function *M* and its distribution.

Let N_f be the number of simulation cycles when g(*X*) is less than zero and let *N* be the total number of simulation cycles. Therefore, an estimate of the probability of failure can be expressed as

(20)

Numerical Example

The probability of failure for the five-storey base-isolated building is evaluated considering the following limit state function:

(21)

where*h*(*X*) is the limit state function associated with the top floor acceleration response of the building and a_i is the peak acceleration of the top floor in g evaluated considering the stochastic nature of the input ground motion.A value of 0.3 g is taken as the allowable acceleration at the top floor for the building under consideration. Then, the probability of failure is defined as

(22)

The probability of failure is evaluated via Monte Carlo simulation by determining the number of realizations with and dividing that number by the total number of simulations. The convergence of the probability of failure is demonstrated in Fig. 5 where the isolator parameters are:, , and. The figure also depicts the probability of failure evaluated using FORM. It is seen that the FORM has also provided a reasonably good approximation as compared to the MCS.

Fig. 5 An example of convergence of probability of failure for the isolation system

CONCLUSIONS

Base isolation is the most powerful tool in earthquake engineering pertaining to the passive structural vibration control technologies. In this study, a database to use in line with the probabilistic ground motion model is created using the 100 recorded earthquake motions. Using the ground motion model and Monte Carlo simulation (MCS), 5000 artificial acceleration-time histories are generated and used to carry out the stochastic response analysis of the five-storey base-isolated building. In order to carry out the seismic reliability analysis, the top floor acceleration response statistics are used in the performance function considering 0.3g as a maximum allowable acceleration and the probability of failure is evaluated. The MCS is also a convenient and accurate method for calculating the probability of failure associated with any of the response statistics. The probability of failure evaluated using the

FORM is fairly in good agreement with the value obtained by the MCS.

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